SWITCHING ALGEBRA

$$\text{Algebra} = \begin{cases} \text{Elements} & = & B = \{0, 1\} \\ \quad + & & \\ \text{Operators} & = & \{\text{OR, AND, NOT}\} \end{cases}$$

Properties of Boolean algebra

1a. $x + 0 = x$	1b. $x \cdot 1 = x$	Identity element
2a. $x + 1 = 1$	2b. $x \cdot 0 = 0$	Universal bound
3a. $x + x = x$	3b. $x \cdot x = x$	Idempotency
4a. $x + \bar{x} = 1$	4b. $x \cdot \bar{x} = 0$	Complementation
5. $\bar{\bar{x}} = x$		Involution

6a. $x + y = y + x$	Commutativity
6b. $x \cdot y = y \cdot x$	
7a. $x + (y + z) = (x + y) + z$	Associativity
7b. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$	
8a. $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$	Distributivity
8b. $x + (y \cdot z) = (x + y) \cdot (x + z)$	
9a. $\overline{x + y} = \bar{x} \cdot \bar{y}$	DeMorgan's law
9b. $\overline{x \cdot y} = \bar{x} + \bar{y}$	
10a. $x + (x \cdot y) = x$	Absorption
10b. $x \cdot (x + y) = x$	
11a. $(x \cdot y) + (x \cdot \bar{y}) = x$	Combining
11b. $(x + y) \cdot (x + \bar{y}) = x$	
12a. $x + y = x + z \Rightarrow y = z$	Non-Cancellation
12b. $x \cdot y = x \cdot z \Rightarrow y = z$	
13a. $x + (\bar{x} \cdot y) = x + y$	No name?
13b. $x \cdot (\bar{x} + y) = x \cdot y$	

Where $+ =$ OR, $\cdot =$ AND, $\bar{\ } =$ NOT

Switching Functions $f\colon \{0,1\}^n \mapsto \{0,1\}$

f is a function of n variables $x_1, \ldots, x_n$ such that

$$\begin{cases} x_i & \in \{0,1\},\ 1 \le i \le n \\ f(x_1,\ldots,x_n) & \in \{0,1\} \end{cases}$$

Any Boolean function f can be specified either by

Truth table

Row#		x_1	x_2	$\ldots$	x_{n-1}	x_n	f
0	$\mapsto$	0	0	$\ldots$	0	0	-
1	$\mapsto$	0	0	$\ldots$	0	1	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$2^n - 2$	$\mapsto$	1	1	$\ldots$	1	0	-
$2^n - 1$	$\mapsto$	1	1	$\ldots$	1	1	-

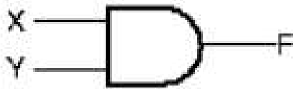
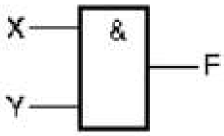
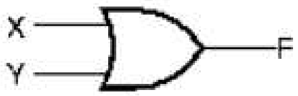
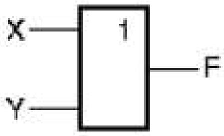
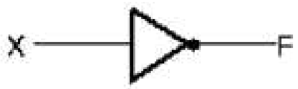
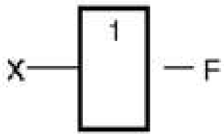
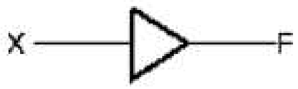
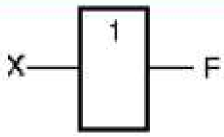
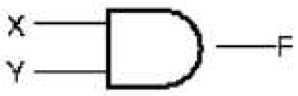
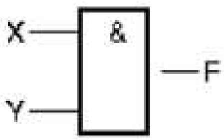
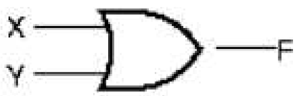
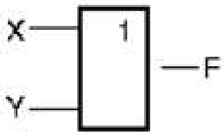
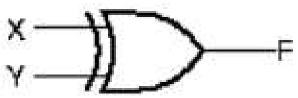
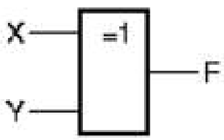
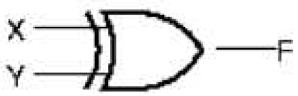
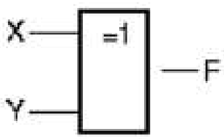
There are 2^n combinations of 0's and 1's.

Boolean expression e such that $f(x_1,\ldots,x_n) = e$ where e is either a Boolean constant, a Boolean variable, or any expression that contains constants, variables or Boolean operators. Examples:

$$f(x_1,x_2,x_3) = (x_1 \oplus x_3) \cdot \bar{x}_2 + 1$$
$$f(x_1,x_2) = x_1$$
$$f(x_1,x_2,x_3,x_4) = 0$$

Boolean algebra is used to simplify and minimize Boolean expressions for given Boolean functions

Basic Switching Functions
(Logic Gates)

Name	Distinctive shape	Rectangular shape	Algebraic equation	Truth table
AND			$F = XY$	X Y F 0 0 0 0 1 0 1 0 0 1 1 1
OR			$F = X + Y$	X Y F 0 0 0 0 1 1 1 0 1 1 1 1
NOT (inverter)			$F = \overline{X}$	X F 0 1 1 0
Buffer			$F = X$	X F 0 0 1 1
NAND			$F = \overline{X \cdot Y}$	X Y F 0 0 1 0 1 1 1 0 1 1 1 0
NOR			$F = \overline{X + Y}$	X Y F 0 0 1 0 1 0 1 0 0 1 1 0
Exclusive–OR (XOR)			$F = X\overline{Y} + \overline{X}Y$ $= X \oplus Y$	X Y F 0 0 0 0 1 1 1 0 1 1 1 0
Exclusive–NOR (XNOR)			$F = XY + \overline{X}\overline{Y}$ $= \overline{X \oplus Y}$	X Y F 0 0 1 0 1 0 1 0 0 1 1 1

Algebraic Simplifications

(Reduce the length of a Boolean expression)

Simplify $f(x_1, x_2, x_3) = x_1\bar{x}_3 + \bar{x}_1 x_2 + \bar{x}_1\bar{x}_2\bar{x}_3$

$x_1\bar{x}_3 + \underbrace{\bar{x}_1 x_2 + \bar{x}_1 x_2\bar{x}_3} + \bar{x}_1\bar{x}_2\bar{x}_3$	Absorption
$x_1\bar{x}_3 + \bar{x}_1 x_2 + \underbrace{\bar{x}_1\bar{x}_3(x_2 + \bar{x}_2)}$	Distributivity
$x_1\bar{x}_3 + \bar{x}_1 x_2 + \bar{x}_1\bar{x}_3\,\underbrace{(1)}$	Complementation
$x_1\bar{x}_3 + \bar{x}_1 x_2 + \underbrace{\bar{x}_1\bar{x}_3}$	Identity element
$\underbrace{\bar{x}_1 x_2 + x_1\bar{x}_3} + \bar{x}_1\bar{x}_3$	Commutativity
$\bar{x}_1 x_2 + \underbrace{\bar{x}_3 x_1 + \bar{x}_3\bar{x}_1}$	Commutativity
$\bar{x}_1 x_2 + \bar{x}_3\,\underbrace{(x_1 + \bar{x}_1)}$	Distributivity
$\bar{x}_1 x_2 + \bar{x}_3\,\underbrace{(1)}$	Complementation
$\Rightarrow f(x_1, x_2, x_3) = \bar{x}_1 x_2 + \underbrace{\bar{x}_3}$	Identity element

Simplify $f(x_1, x_2, x_3) = x_1\bar{x}_3 + \bar{x}_2\bar{x}_3 + x_1 x_3 + \bar{x}_2 x_3$

$x_1\bar{x}_3 + \underbrace{x_1 x_3 + \bar{x}_2\bar{x}_3} + \bar{x}_2 x_3$	Commutativity
$\underbrace{x_1(\bar{x}_3 + x_3)} + \underbrace{\bar{x}_2(\bar{x}_3 + x_3)}$	Distributivity
$x_1\,\underbrace{(1)} + \bar{x}_2\,\underbrace{(1)}$	Complementation
$\Rightarrow f(x_1, x_2, x_3) = \underbrace{x_1} + \underbrace{\bar{x}_2}$	Identity element

Simplify

$$f(x_1, x_2) = \bar{x}_1\bar{x}_2 + x_1 x_2 + x_1\bar{x}_2$$
$$f(x_1, x_2, x_3) = x_1 + x_1 x_2 x_3 + \bar{x}_1 x_3$$
$$f(x_1, x_2, x_3) = (x_1 + \bar{x}_2 + x_1\bar{x}_2)(x_1 x_2 + \bar{x}_1 x_3 + x_2 x_3)$$
$$f(x_1, x_2, x_3, x_4) = (x_2\bar{x}_3 + \bar{x}_1 x_4)(x_1\bar{x}_2 + x_3\bar{x}_4)$$

Demonstrations

(Are two Boolean expressions equivalent?)

Algebraic demonstration

$$(x_1 + x_3)(\bar{x}_1 + \bar{x}_3) \stackrel{?}{=} x_1\bar{x}_3 + \bar{x}_1 x_3$$

LHS $=$	
$(x_1 + x_3)\bar{x}_1 + (x_1 + x_3)\bar{x}_3$	Distributivity
$x_1\bar{x}_1 + x_3\bar{x}_1 + x_1\bar{x}_3 + x_3\bar{x}_3$	Distributivity
$0 + x_3\bar{x}_1 + x_1\bar{x}_3 + 0$	Complementation
$x_3\bar{x}_1 + x_1\bar{x}_3$	Identity element
$x_1\bar{x}_3 + \bar{x}_1 x_3$	Commutativity
$=$ RHS	

Tabular demonstration

$$\overline{x \cdot y} \stackrel{?}{=} \bar{x} + \bar{y}$$

x	y	$\bar{x}$	$\bar{y}$	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x} + \bar{y}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	1	1	0	0
					LHS	RHS

What are the advantages of algebraic demonstration over tabular demonstration and vice-versa?

Minterms and Maxterms

In truth table of $f(x_1, \ldots, x_n)$ each combination of 0's and 1's can be represented as a

Minterm $l_1 \cdot l_2 \cdot \ \cdots \ \cdot l_n$ (product term)
$= 0$ if a least one literal is 0

Maxterm $l_1 + l_2 + \ \cdots \ + l_n$ (sum term)
$= 1$ if a least one literal is 1

where *literal* $l_i = x_i$ or $\bar{x}_i$, for $1 \leq i \leq n$

3-variable minterms and maxterms

Row	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 : \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 : x_1 + x_2 + x_3$
1	0	0	1	$m_1 : \bar{x}_1\bar{x}_2 x_3$	$M_1 : x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 : \bar{x}_1 x_2\bar{x}_3$	$M_2 : x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 : \bar{x}_1 x_2 x_3$	$M_3 : x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 : x_1\bar{x}_2\bar{x}_3$	$M_4 : \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 : x_1\bar{x}_2 x_3$	$M_5 : \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 : x_1 x_2\bar{x}_3$	$M_6 : \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 : x_1 x_2 x_3$	$M_7 : \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

DeMorgan's law $\Rightarrow \begin{cases} \bar{m}_i & = & M_i \\ \bar{M}_i & = & m_i \end{cases}$ for $1 \leq i \leq n$

Note: not all product terms (respectively, sum terms) are minterms (respectively, maxterms)

Canonical Forms

Any function $f(x_1, \ldots, x_n)$ can be expressed as a

Canonical sum of products: (CSOP) Sum of minterms for which f has value 1

$$f(x_1, \ldots, x_n) = \sum_{i=0}^{2^n-1} (\alpha_i \cdot m_i)$$

Canonical product of sums: (CPOS) Product of maxterms for which f has value 0

$$f(x_1, \ldots, x_n) = \prod_{i=0}^{2^n-1} (\alpha_i + M_i)$$

where α_i $(1 \leq i \leq 2^n - 1)$ is the value of f

Example if $f(x_1, x_2) = x_1 \oplus x_2$ then

CSOP: $f(x_1, x_2) = m_1 + m_2 = \sum m(1, 2)$

$$0\bar{x}_1\bar{x}_2 + 1\,\underbrace{\bar{x}_1 x_2}_{m_1} + 1\,\underbrace{x_1\bar{x}_2}_{m_2} + 0x_1x_2$$

CPOS: $f(x_1, x_2) = M_0 \cdot M_3 = \prod M(0, 3)$

$$(0 + \underbrace{x_1 + x_2}_{M_0})(1 + x_1 + \bar{x}_2)(1 + \bar{x}_1 + x_2)(0 + \underbrace{\bar{x}_1 + \bar{x}_2}_{M_3})$$

Examples of Canonical Forms

$$f(x_1, x_2, x_3) = \overline{x_1 \oplus x_2 \oplus x_3}$$

#	x_1	x_2	x_3	Minterm	Maxterm	f
0	0	0	0	$\bar{x}_1\bar{x}_2\bar{x}_3$	$x_1 + x_2 + x_3$	1
1	0	0	1	$\bar{x}_1\bar{x}_2x_3$	$x_1 + x_2 + \bar{x}_3$	0
2	0	1	0	$\bar{x}_1x_2\bar{x}_3$	$x_1 + \bar{x}_2 + x_3$	0
3	0	1	1	$\bar{x}_1x_2x_3$	$x_1 + \bar{x}_2 + \bar{x}_3$	1
4	1	0	0	$x_1\bar{x}_2\bar{x}_3$	$\bar{x}_1 + x_2 + x_3$	0
5	1	0	1	$x_1\bar{x}_2x_3$	$\bar{x}_1 + x_2 + \bar{x}_3$	1
6	1	1	0	$x_1x_2\bar{x}_3$	$\bar{x}_1 + \bar{x}_2 + x_3$	1
7	1	1	1	$x_1x_2x_3$	$\bar{x}_1 + \bar{x}_2 + \bar{x}_3$	0

$$\Downarrow$$

CSOP:

$$f(x_1, x_2, x_3) = m_0 + m_3 + m_5 + m_6 = \sum m(0, 3, 5, 6) =$$
$$\bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1x_2x_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

CPOS:

$$f(x_1, x_2, x_3) = M_1 \cdot M_2 \cdot M_4 \cdot M_7 = \prod M(1, 2, 4, 7) =$$
$$(x_1 + x_2 + \bar{x}_3)(x_1 + \bar{x}_2 + x_3)(\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$$

Equivalence of forms CSOP $\leftrightarrows$ CPOS conversions

1. Interchange symbols $\sum$ and $\prod$
2. List those numbers missing in the original

Example from above

$$\sum m(0, 3, 5, 6) = \prod M(1, 2, 4, 7)$$

Standard Forms

Sum of products: (SOP) Not all product terms are minterms
$f(x_1, x_2, x_3) = \bar{x}_1 x_2 + \bar{x}_3 + x_1 x_2 x_3$

Product of sums: (POS) Not all sum terms are maxterms
$f(x_1, x_2, x_3) = (\bar{x}_1)(\bar{x}_2 + x_3)$

From standard form to canonical form

SOP $\mapsto$ CSOP For each missing variable x_i in a term t perform $(x_i + \bar{x}_i) \cdot t$. Then simplify and eliminate redundant minterms

Ex: $f(x_1, x_2, x_3) = \bar{x}_1 x_2 + \bar{x}_3 + x_1 x_2 x_3$
$= \bar{x}_1 x_2 (x_3 + \bar{x}_3) + (x_1 + \bar{x}_1)(x_2 + \bar{x}_2)\bar{x}_3 + x_1 x_2 x_3$
$= \bar{x}_1 x_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$
$= \bar{x}_1 x_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$
$= m_3 + m_2 + m_6 + m_4 + m_0 + m_7$
$= \sum m(0, 2, 3, 4, 6, 7) =$ CSOP $f(x_1, x_2, x_3)$

POS $\mapsto$ CPOS For each missing variable x_i in a term t perform $(x_i \cdot \bar{x}_i) + t$. Then simplify and eliminate redundant maxterms

Remarks on Logic Functions

Uniqueness The CSOP or CPOS of a logic function is unique

Equivalence $f_1 = f_2 \Leftrightarrow \begin{cases} \text{CSOP } f_1 = \text{CSOP } f_2 \\ \text{or} \\ \text{CPOS } f_1 = \text{CPOS } f_2 \end{cases}$

Number of n-variable switching functions 2^{2^n}

Ex: there are 2^{2^2} 2-variable switching functions

Truth tables of 16 functions of two variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7
0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1
Op.			$\cdot$	$/$		$/$		$\oplus$	$+$

x	y	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1
Op.		$\downarrow$	$\sim$	$\bar{\ }$	$\rightarrow$	$\bar{\ }$	$\rightarrow$	$\uparrow$	

Two-Variable Switching Functions

Function	Op.	Name	Comment
$F_0 = 0$		Null	constant 0
$F_1 = xy$	$x \cdot y$	AND	x and y
$F_2 = x\bar{y}$	x/y	Inhibition	x but not y
$F_3 = x$		Transfer	x
$F_4 = \bar{x}y$	y/x	Inhibition	y but not x
$F_5 = y$		Transfer	y
$F_6 = x\bar{y} + \bar{x}y$	$x \oplus y$	Exclusive-OR	either x or y
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = \overline{x + y}$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + \bar{x}\bar{y}$	$x \sim y$	Exclusive-NOR	x equals y
$F_{10} = \bar{y}$	$\bar{y}$	Complement	Not y
$F_{11} = x + \bar{y}$	$y \rightarrow x$	Implication	if y then x
$F_{12} = \bar{x}$	$\bar{x}$	Complement	Not x
$F_{13} = \bar{x} + y$	$x \rightarrow y$	Implication	if x then y
$F_{14} = \overline{xy}$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	constant 1

Exclusive-NOR is also known as *Equality*, *Coincidence* or *Equivalence*

From canonical form to standard form

- By algebraic simplification (as in slide #4)
- By graphical simplification (we'll see soon)

Functional Completeness

A set F of operators is *functionally complete* if and only if any Boolean function $f\colon \{0,1\}^n \mapsto \{0,1\}$ can be expressed using operators from F only

$F = \{$**OR,AND,NOT**$\}$ is functionally complete (?)

$F = \{$**NOR**$\}$ is functionally complete

OR : $x + y = \overline{\overline{x+y}} = \overline{\overline{x+y}+0} = (x \downarrow y) \downarrow 0$

AND : $x \cdot y = \overline{\bar{x} + \bar{y}} = (x \downarrow 0) \downarrow (y \downarrow 0)$

NOT : $\bar{x} = \overline{x+0} = x \downarrow 0$

$F = \{$**NAND**$\}$ is functionally complete

OR : $x + y = \overline{\bar{x} \cdot \bar{y}} = (x \uparrow 1) \uparrow (y \uparrow 1)$

AND : $x \cdot y = \overline{\overline{x \cdot y}} = \overline{\overline{x \cdot y} \cdot 1} = (x \uparrow y) \uparrow 1$

NOT : $\bar{x} = \overline{x \cdot 1} = x \uparrow 1$

Example

$$
\begin{aligned}
f(x,y,z) &= (x+y)(\bar{y}+z) = \overline{\overline{x+y} + \overline{\bar{y}+z}} \\
&= (x \downarrow y) \downarrow (\bar{y} \downarrow z)
\end{aligned}
$$

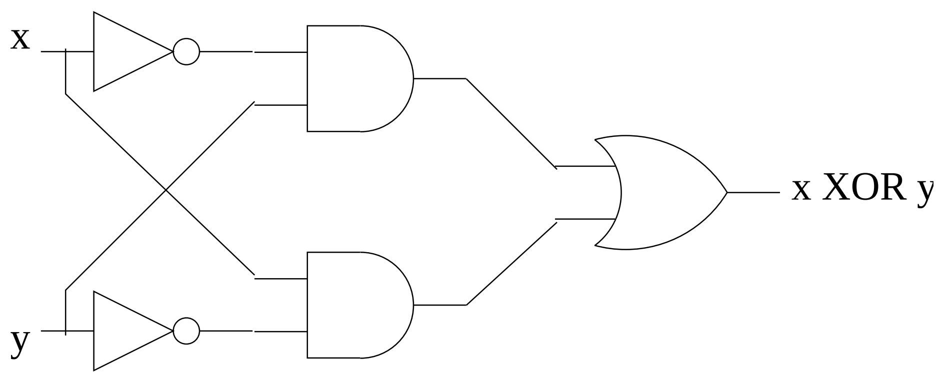
$$
f(x,y,z) = xy + \bar{y}z = \overline{\overline{xy} \cdot \overline{\bar{y}z}} = (x \uparrow y) \uparrow (\bar{y} \uparrow z)
$$

Switching Circuits

Implement (or realize) switching functions

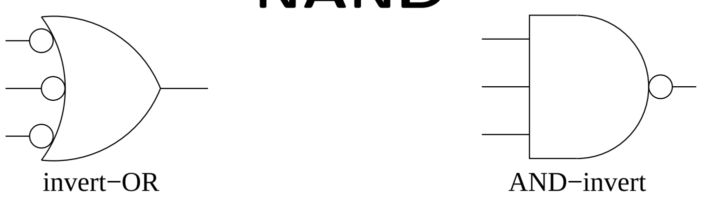
- Expression ↦ Circuit
- Term ↦ Gate
- Literal ↦ Input to gates

Functions can be realized in *many ways*

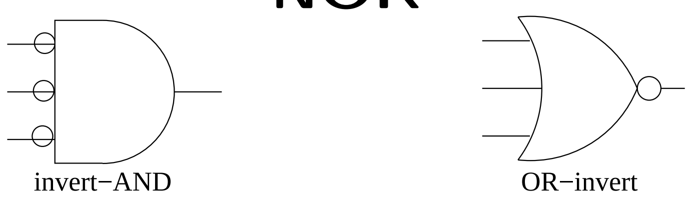


Some gates can have *more than 2* inputs

NAND



NOR

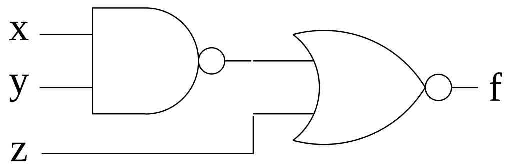


Logic Design and Analysis

Design Realize the logic circuit corresponding to a given function

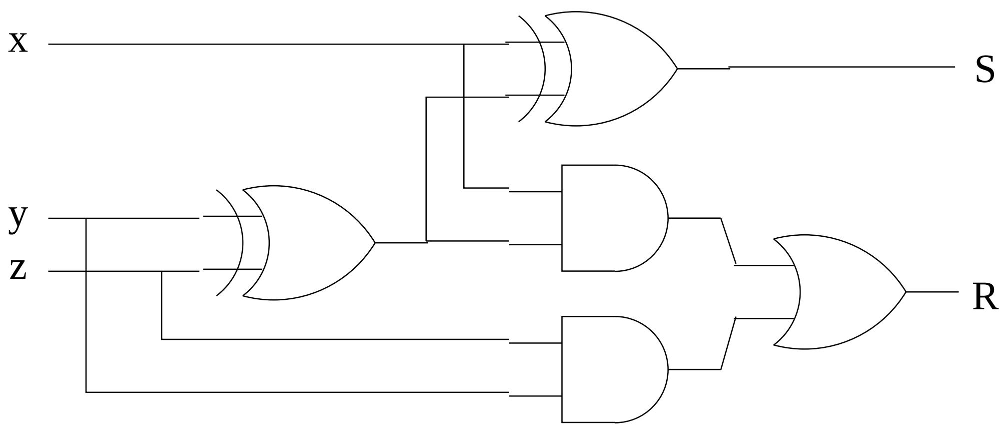
$$f(x,y,z) \quad = \quad \overline{\overline{xy}+z}$$

$$\Downarrow$$



The circuit may be further minimized if necessary

Analysis Determine the logic function corresponding to a given circuit



$$\Downarrow$$

$$f(x,y,x) = (S,R) \quad = \quad (x \oplus y \oplus z, x(y \oplus z) + yz)$$

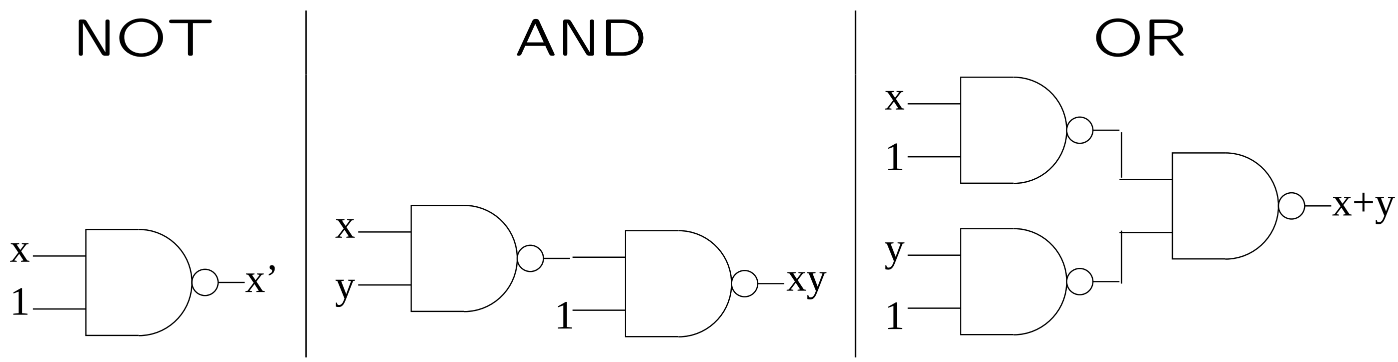
The function may be further minimized if necessary

Circuits with Universal Gates

NAND and NOR gates are *simpler* and *cheaper* than NOT, AND, OR gates

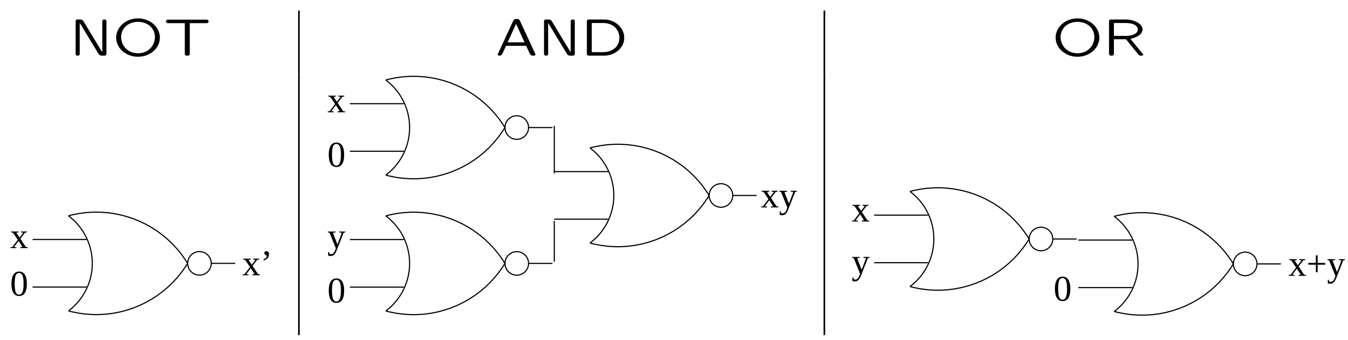
NAND Circuits

(Better for implementing SOP expressions)



NOR Circuits

(Better for implementing POS expressions)



Implementations with NAND Gates

SOP expression

$$
\begin{aligned}
f(a,b,c,d) &= a\bar{b} + \bar{a}c + d \\
&= \overline{\overline{a\bar{b} + \bar{a}c + d}} \\
&= \overline{\overline{a\bar{b}} \cdot \overline{\bar{a}c} \cdot \bar{d}} \\
&= \overline{a\bar{b}} \uparrow \overline{\bar{a}c} \uparrow \bar{d} \\
&= (a \uparrow \bar{b}) \uparrow (\bar{a} \uparrow c) \uparrow \bar{d} \\
&= (a \uparrow (b \uparrow 1)) \uparrow ((a \uparrow 1) \uparrow c) \uparrow (d \uparrow 1)
\end{aligned}
$$

POS expression

$$
\begin{aligned}
g(a,b,c,d) &= (a + \bar{b})(\bar{a} + c)d \\
&= \overline{\overline{a + \bar{b}}} \cdot \overline{\overline{\bar{a} + c}} \cdot d \\
&= \overline{\bar{a}b} \cdot \overline{a\bar{c}} \cdot d \\
&= (\bar{a} \uparrow b)(a \uparrow \bar{c})d \\
&= \overline{\overline{(\bar{a} \uparrow b)(a \uparrow \bar{c})d}} \\
&= \overline{(\bar{a} \uparrow b) \uparrow (a \uparrow \bar{c}) \uparrow d} \\
&= ((\bar{a} \uparrow b) \uparrow (a \uparrow \bar{c}) \uparrow d) \uparrow 1 \\
&= (((a \uparrow 1) \uparrow b) \uparrow (a \uparrow (c \uparrow 1)) \uparrow d) \uparrow 1
\end{aligned}
$$

Implementations with NOR Gates

SOP expression

$$
\begin{aligned}
f(a,b,c,d) &= a\bar{b} + \bar{a}c + d \\
&= \overline{\overline{a\bar{b}}} + \overline{\overline{\bar{a}c}} + d \\
&= \overline{\bar{a} + b} + \overline{a + \bar{c}} + d \\
&= (\bar{a} \downarrow b) + (a \downarrow \bar{c}) + d \\
&= \overline{\overline{(\bar{a} \downarrow b) + (a \downarrow \bar{c}) + d}} \\
&= \overline{(\bar{a} \downarrow b) \downarrow (a \downarrow \bar{c}) \downarrow d} \\
&= ((\bar{a} \downarrow b) \downarrow (a \downarrow \bar{c}) \downarrow d) \downarrow 0 \\
&= (((a \downarrow 0) \downarrow b) \downarrow (a \downarrow (c \downarrow 0)) \downarrow d) \downarrow 0
\end{aligned}
$$

POS expression

$$
\begin{aligned}
g(a,b,c,d) &= (a + \bar{b})(\bar{a} + c)d \\
&= \overline{\overline{(a + \bar{b})(\bar{a} + c)d}} \\
&= \overline{\overline{a + \bar{b}} + \overline{\bar{a} + c} + \bar{d}} \\
&= \overline{a + \bar{b}} \downarrow \overline{\bar{a} + c} \downarrow \bar{d} \\
&= (a \downarrow \bar{b}) \downarrow (\bar{a} \downarrow c) \downarrow \bar{d} \\
&= (a \downarrow (b \downarrow 0)) \downarrow ((a \downarrow 0) \downarrow c) \downarrow (d \downarrow 0)
\end{aligned}
$$